# Could zero point energy be gravitational energy? Another point of view of the cosmological constant problem.

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A B S T R A C T

The cosmological constant problem or vacuum catastrophe is localized in the convergence between general relativity and quantum field theory, it is considered as a fundamental problem in modern physics. In this paper we describe a different point of view of this problem. We discuss problem could depend to different definition of the vacuum energy density.

**Introduction**

Quantum field theory (QFT) which is fundamental in modern physics show zero-point energy in space, including in areas which are in another way 'void' (i.e. without radiation and matter). Maybe we could think these zero-point energy give a vast vacuum energy density. On the other hand it is expect to cause an increase of cosmological constant $\Lambda$ appearing in Einstein's field equation.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (1)$$

Where $R_{\mu\nu}$ and R allude the curvature of space-time, $g_{\mu\nu}$ is the metric, G is the gravitational constant, $T_{\mu\nu}$ is the stress-energy tensor, and c is the speed of light. ( Zinkernagel & Rugh 2000).

Equations should be agree with Newtonian theory in the limit for medium and small gravitational fields and small velocities, and this should be consonant also with the value of $\Lambda$ ( Zinkernagel & Rugh 2000). In fact, compare equation (1) with observations show that $\Lambda$ is minuscule.

$$|\Lambda| < 10^{-56}\text{cm}^2 \qquad (2)$$

The Friedmann equations are a set of equations in physical cosmology that govern the expansion of space in homogeneous and isotropic models of the universe based on general relativity.(Friedman, A. 1999).There are two independent Friedmann equations, the first is:

$$H_0{}^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} + \frac{kc^2}{a^2} \quad (3)$$

Where:

$$\rho_m + \rho_r = \rho \quad (4)$$

$$\rho_{Vac} = \rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad (5)$$

$H_0$ is the Hubble constant.

$a$ is the scale factor.

k is the intrinsic curvature.

$\Lambda$ is the cosmological constant.

$\rho_R$ is radiation energy density divided by the speed of light squared.

$\rho_M$ is the matter (dark plus baryonic) energy density divided by the speed of light squared.

$\rho_{Vac}$ is the vacuum energy density divided by the speed of light squared.

$\rho_\Lambda$ is the cosmological constant energy density divided by the speed of light squared, it represent cosmological constant effect.

Vacuum energy is the background energy that exists in the universe. The effect of the vacuum energy appears in the first Friedmann equation, vacuum energy is expected to create the cosmological constant, and produce the expansion of the universe.

If $\rho = 0$ and setting the normalized spatial curvature,k, equal to zero according to the $\Lambda$CDM model when the substitutions are applied to the first of the Friedmann´s equations we find:

$$\rho_{Vac} = \rho_\Lambda = \frac{3H_0{}^2}{8\pi G} \sim 10^{-29}\frac{g}{cm^3} \mapsto \rho_{Vac}c^2 = \rho_\Lambda c^2 = \frac{3H_0{}^2c^2}{8\pi G} \sim 10^{-9}\frac{erg}{cm^3}$$

This equation usually has been understood as the vacuum energy density in gravitational theory, nevertheless, theoretical estimates the vacuum energy density in QFT exceed the observational measurement by at least 40 orders of magnitude. This contrast constitutes the cosmological constant problem ( Zinkernagel & Rugh 2000).

Quantum electrodynamics is one of the principal components in the Standard Model, first, and most productive case of a working quantum field theory. We first remembrance that systems studied in non-relativistic quantum mechanics, spatial-temporal coordinates, energy and others are represented by operators in quantum electrodynamics

( Zinkernagel & Rugh 2000).

One of the most simple quantum systems is the quantum harmonic oscillator. The ground state (vacuum state) of the quantum harmonic oscillator has zero point energy:

$$E_0 = \frac{1}{2}\hbar\omega_k \quad (6)$$

(where $\omega_k$ is the frequency of classical harmonic oscillator).

In classical electromagnetism, electromagnetic fields have values E, and B in all space-time.

The energy density in the classical electromagnetism theory is:

$$H = \frac{1}{2}(E^{-2} + B^{-2}) \quad (7)$$

In the quantization, quantum operators take the place of classical fields, The ground state $|0>$ is the vacuum state of QED.

Whole zero-point energy of the quantum electrodynamic theory can be expressed by:

$$E = \ < 0|\, \hat{\mathbf{H}}_{\mathbf{i}}|0 > \qquad (8)$$

$$E = \frac{1}{2} < 0|\left(\hat{\mathbf{E}}_{\mathbf{i}}^{-2} + \hat{\mathbf{B}}_{\mathbf{i}}^{-2}\right)|0 >= \delta(0)^3 \int d^3k \frac{1}{2}\hbar\omega_k \qquad (9)$$

Where $\omega_k$ are wave-numbers and k are frequencies of a continuum of (plane-wave) modes.

The infinite delta-function $\delta^3(0)$ can be regularized introducing a cube of volume V.

This volumes (in the limit $V \to \infty$) represent the standard 'box-quantization' process for the electromagnetic field in which an artificial 'quantization volume' V is used to create an equivalence with a harmonic quantum oscillator field mode. Energy density in this approximation,

(when $\rho_\Lambda = \rho_{Vac}$) can be extract from the zero-point energy of harmonic quantum oscillator mode:

$$\rho_{Vac}c^2 = \frac{E}{v_i} = \frac{1}{V}\sum_k \frac{1}{2}\hbar\omega_k = \frac{\hbar}{2\pi^2 c^3}\int_0^{\omega_{max}} \omega^3\, d\omega$$

$$\rho_{Vac}c^2 = \frac{\hbar}{8\pi^2 c^3}{\omega_{max}}^4 \qquad (10)$$

Where the wave vector k now refers to the so-called normal modes (of the electromagnetic field) which are compatible with the boundary conditions provided by the box volume V. The right hand side of the equation follows from the left hand side in the limit $V \to \infty$ where the energy density does not depend on the 'box quantization' volume V. If we imagine that the QFT framework is valid to the Planck energy:

$$E_{planck} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} = 10^{19}\ \text{GeV} \qquad (11)$$

It is easy to see, modern physics could enter between electroweak scales and Planck scales but if the alternations are still out of the framework of quantum field theory, we could admit vacuum energy can be expressed (using Planck energy in eq.(12) with $E_{Planck} = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} = \hbar\omega_{max}$ ):

$$\rho_{Vac}c^2 = 10^{114}\,\frac{erg}{cm^3} \qquad (12)$$

There are a difference between QED and observational result by more than ~ 120 orders of magnitude ( Zinkernagel & Rugh 2000).

**Proposition**

On the one hand we introduce Hubble´s radius:

$$R = \frac{c}{H_0} \qquad (13)$$

Hubble´s radius (observable universe radius) is a theoretical horizon defining the limit between particles that are moving slower and faster than the speed of light. In the current expansion model FWR light emitted from the Hubble´s radius will reach us in a finite amount of time, this is observable universe limit (Earman & Moster´ın 1999; Heitler 1954).

Hubble´s radius will be our universe radius.

On the other hand we use a similar Planck energy equation in the minimum wavelength:

$$E_{\omega_{max}} = \left(\frac{hc^5}{G}\right)^{\frac{1}{2}} \qquad (14)$$

First we calculate vacuum energy of one line, vacuum energy along R is the sum of vacuum states of three-dimensional harmonic oscillators along R.

We compare vacuum states sum along R with continuous one-dimensional wave equation. (We calculate it in two different ways and we take wave properties of the QED vacuum).

First way, standing waves in a cavity at equilibrium with its surroundings can't take just any path. They must satisfy the wave equation:

$$\frac{\partial^2 A}{\partial x^2} = \frac{\partial^2 A}{c\,\partial t^2} \qquad (15)$$

The solution to the wave equation must give zero amplitude at the walls:

$$A = A_0 \sin\frac{n\pi x}{L} \sin\frac{2\pi ct}{\lambda} \qquad (16)$$

Substituting this solution into the wave equation above gives:

$$\frac{n\pi}{L} = \frac{2\pi}{\lambda} \qquad (17)$$

Which simplifies to:

$$n = \frac{2L}{\lambda} \qquad (18)$$

We need to evaluate the number of modes $n_{Mod}$ which can meet this condition, which amounts to counting all the possible combinations of the integer n values. An approximation can be made by treating the number of combinations as line grid of the values of n, an "n -space" with the n values specifying the coordinates along one " n " line (Department of physics and astronomy Georgia State University):

$$n_{Mod} = n = \frac{2L}{\lambda} \qquad (19)$$

We calculate vacuum energy alongL:

$$\sum E_{ground} = \frac{1}{2}\hbar\overline{\omega_k}\frac{2L}{\bar{\lambda}} = \frac{L}{2\pi c}\hbar\overline{\omega_k}^2 \qquad (20)$$

If we want calculate medium value of $\omega_k$ we will take oscillators frequencies from $\omega_{min} \sim 0$ to $\omega_{max} = \omega_{Planck}$, therefore $\overline{\omega_k} = \omega_{max}/2$:

$$\sum E_{ground} = \frac{L\hbar\omega_{max}^2}{8\pi c} \qquad (21)$$

Now we calculate it in the limit $L \to R$:

$$\sum E_{ground} = \frac{R\hbar\omega_{max}^2}{8\pi c} \qquad (22)$$

Using (14) we find:

$$\sum E_{ground} = \frac{Rc^4}{4G} \qquad (23)$$

We expose another way to obtain the same approximation.

The range of oscillators frequencies is between $\omega_{min} \sim 0$ and $\omega_{max} = \omega_{Planck}$, therefore vacuum energy along R is:

$$\sum E_{ground} \sim \delta(0) \int dk \frac{1}{2}\hbar\omega_k \qquad (24)$$

Where k are frequencies of a continuum of (wave) modes.

The infinite delta-function $\delta(0)$ can be regularized in a line of length L. "Line regularization" (enclosing energy in a line of length L) implies the following substitution:

$$\delta(0) = \frac{1}{2\pi}\int e^{ikx} dx \to \frac{L}{2\pi} \quad \text{if} \quad k \to 0 \qquad (25)$$

This L length (in the limit $L \to R \sim \infty$ ) represent the standard 'L - quantization' process. Where the wave vector k now refers to the so-called normal modes which are compatible with the boundary conditions provided by the line L. The right hand side of the equation follows from the left hand side in the limit $L \to \infty$ where the energy does not depend on the 'line quantization':

$$\sum E_{ground} \sim \frac{L\hbar}{4\pi c}\int_0^{\omega_{max}} \omega\, d\omega = \frac{L\hbar\omega_{max}^2}{8\pi c} \qquad (26)$$

If $L \to R$ and using (14) and (26) we find:

$$\sum E_{ground} = \frac{Rc^4}{4G} \qquad (27)$$

The vacuum catastrophe could be produced by an error in the definition of $\rho_\Lambda$, in this model $\rho_{Vac} \neq \rho_\Lambda$.

If $\rho = 0$ and setting the normalized spatial curvature,k, equal to zero according to the ΛCDM model we obtain:

$$\rho_\Lambda = \frac{3{H_0}^2}{8\pi G} \qquad (28)$$

If we consider a spherical universe of radius R which expands, mass inside the sphere $M_\Lambda$ can be expressed:

$$\rho_\Lambda = \frac{3M_\Lambda}{4\pi R^3} \qquad (29)$$

Using(13), (28) and (29) we obtain:

$$c^2 = \frac{2GM_\Lambda}{R} \qquad (30)$$

Let's assume that energy of vacuum has the structure of the gravitational energy, Let's assume that energy of vacuum is produced by the interaction between a mass in the center of the universe and a mass located outside the universe, if we consider the energy of a single-axis vacuum along the radius of the universe can be represented as the gravitational energy needed to the mass in the center of the universe could attract mass located outside the universe where product of both masses are ${M_\Lambda}^2$, we will obtain:

$$\sum E_{ground} = \frac{Rc^4}{4G} = \frac{G{M_\Lambda}^2}{R} \qquad (31)$$

If we take (13), (28), (29), (30), (31)

we obtain:

$$\rho_\Lambda = \frac{3M_\Lambda}{4\pi R^3} = \frac{3{H_0}^2}{8\pi G} \qquad (32)$$

**Conclusions**

The observed value of the cosmological constant poses large theoretical problems. We expose a point of view to the vacuum catastrophe, we use QED and ΛCDM model to build this theory. Cosmological constant problem could be caused by definition of $\rho_\Lambda$, the error was replaced $\rho_\Lambda$ by $\rho_{Vac}$, difference between all vacuum energy of the universe$\rho_{Vac}$ and only one line of vacuum energy $\rho_\Lambda$, difference between mass-energy equivalence and gravitational energy could produce the vacuum catastrophe.

**References**


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